Question 1. **Marks**

(a) Evaluate $\dfrac{e^3 - 2\cdot1^2}{\sqrt{3\cdot14 + 2\cdot1}}$ correct to 2 significant figures. **1**

(b) Given that $\tan\theta = \dfrac{7}{8}$ and $\cos\theta < 0$, find the exact value of cosecθ. **2**

(c) Solve for x: $|4x - 15| \le 3$. **2**

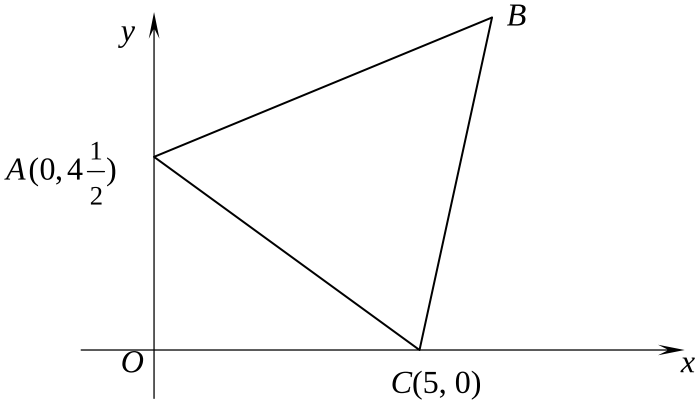
(d) State the period and the amplitude for the graph of $3y = \sin\left(2x - \dfrac{\pi}{4}\right)$. **2**

(e) Paint at the local hardware store is sold at a profit of 30% on the cost price. If a can of paint is sold for $\$67\cdot60$, find the cost price. **2**

(f) Solve for α: $\tan\alpha = -0\cdot5$, where $0 < \alpha < \pi$, correct to 2 decimal places. **1**

(g) Two fair dice are rolled at random. Find the probability that the two numbers are the two digits of a perfect square? **2**

Question 2. **[START A NEW PAGE]**



not to scale

The lines AB and CB have equations: $x - 2y + 9 = 0$ and $4x - y - 20 = 0$ respectively.

(a) Show that the equation of the line AC is $9x + 10y - 45 = 0$. **2**

(b) Calculate the exact distance AC. **1**

(c) Find the coordinates for point B. **2**

(d) Find the angle of inclination of the line through A and B (to nearest degree). **2**

(e) Calculate the shortest distance from point B to the line AC
Hence find the area of triangle ABC. **2**

(f) Determine the inequalities that define the area bounded by $\Delta\, ABC$. **3**

Question 3. **[START A NEW PAGE]** **Marks**

(a) Find $\frac{dy}{dx}$ for

(i) $y = (1 + \ln x)^2$. **2**

(ii) $y = \frac{\sin x}{e^{3x} + 1}$. **2**

(b) (i) Find $\int (5x - 1)^3 \, dx$. **2**

(ii) Find the value: $\int_0^{\pi} \sec^2 \frac{x}{4} \, dx$. **2**

(c) Given that α and β are the roots of the equation $2x^2 - 6x - 7 = 0$,

(i) Find $\alpha + \beta$. **1**

(ii) Find $\alpha^2 + \beta^2$. **2**

(iii) Find $\alpha^2 - 3\alpha$. **1**

Question 4. **[START A NEW PAGE]**

(a) The good ship Lollypop sails from port A 60 nautical miles due west to port B. It then sails a distance of 50 nautical miles on a bearing of 210^0 T to port C

(i) Draw a diagram to illustrate this information. **1**

(ii) Calculate the distance of C from A (to nearest nautical mile). **2**

(iii) Calculate the bearing of port C from port A. **2**

(b) Evaluate: $\lim_{x \to 16} \frac{x - 16}{\sqrt{x} - 4}$. **2**

(c) Find the equation of the normal to the curve $y = 1 + \ln 2x$ at the point $\left(\frac{e}{2}, 2\right)$. **3**

(d) Find the radius and the centre of the circle whose equation is: **2**

$$4x^2 - 4x + 4y^2 + 24y + 21 = 0.$$

Question 5. **[START A NEW PAGE]** **Marks**

(a) 1 000 tickets are sold in a raffle. First prize is \$1 000, second prize is \$500 and third prize of \$200.
The prize winning tickets are drawn consecutively without replacement where the first ticket wins first prize.

(i) Find the probability that a person buying one ticket in the raffle wins:

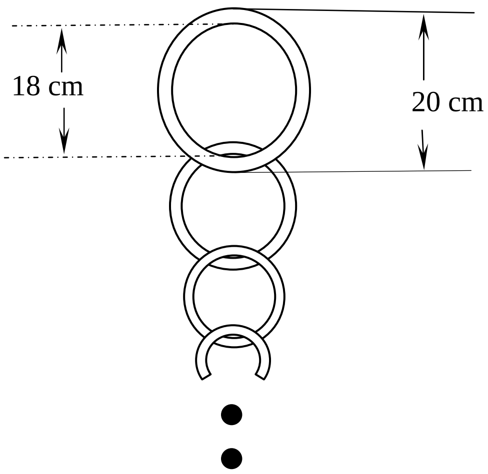
(α) First prize. **1**

(β) at least \$500. **1**

(γ) No prizes. **1**

(ii) A person buying two tickets in the raffle wins at least \$500. **2**

(b) A number of linked rings, each 1 cm thick, are hung from a nail on a wall. The top ring has an outside diameter of 20 cm as shown in the diagram. The outside diameter of each of the other rings is 1 cm less than that of the ring above it. The bottom (last) ring has an outside diameter of 3 cm.

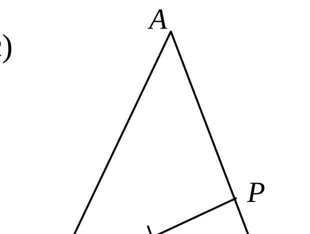


Not to scale

(i) ***Copy*** the diagram and explain why the top of the second ring is 17 cm above the top of the third ring. **1**

(ii) Hence calculate the distance from the top of the top ring to the bottom of the bottom ring **3**

(c)



Not to scale

Given ΔABC is an isosceles triangle with $AB = AC$. **3**
P lies on AC such that $\angle ABP = 3\angle PBC$ and $BP = BC$.

Copy the diagram into your writing booklet and by letting $\angle CBP = x$, or otherwise, find angle $\angle CBP$ expressing it in radians.

Question 6. **[START A NEW PAGE]** **Marks**

(a) Show that $\dfrac{1+\tan^2 A}{\cos ec^2 A} = \tan^2 A.$ **2**

(b) Given the function $y = f(x)$ for $1 \le x \le 2\cdot 5$, where **3**

x	1	$1\cdot 25$	$1\cdot 5$	$1\cdot 75$	2	$2\cdot 25$	$2\cdot 5$
$f(x)$	$3\cdot 4$	$2\cdot 2$	$0\cdot 4$	$1\cdot 9$	$-2\cdot 7$	$1\cdot 3$	2

Use Simpsons' rule to evaluate $\int_1^{2\cdot 5} f(x)\,dx$, correct to 1 decimal place, using the 7 function values in the table.

(c) A function is defined by $y = g(x)$, where $g(x) = x^3 - 6x^2 + 5 = (x-1)(x^2 - 5x - 5)$.

(i) Determine the coordinates of the stationary points and determine their nature. **3**

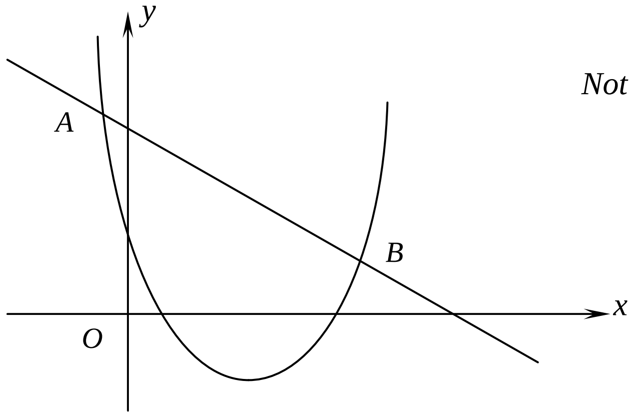
(ii) State at what point does the curve change its concavity? **1**

(iii) Hence sketch the graph of $y = g(x)$, showing all essential details. **2**

(iv) Find the minimum value of $x^3 - 6x^2 + 5$ when $-3 \le x \le 5$. **1**

Question 7. **[START A NEW PAGE]** **Marks**

(a) The parabola $y=(x-3)^2-1$ and the line $y=13-2x$ intersect at points A and B.

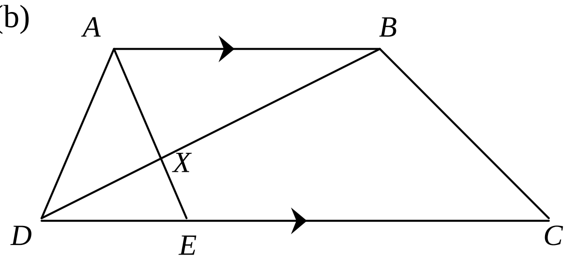


Not to scale

(i) Find the x-coordinate for points A and B. **2**

(ii) Hence, or otherwise find the area bounded by the parabola and the line. **2**

(b)



Not to scale

$ABCD$ is a trapezium where $AB \parallel DC$.
Also $BD \perp BC$ and $AE \perp BD$ at X.

(i) ***Copy*** the diagram into your writing booklet and find AX, given $AD = 41\,\text{cm}$, $DX = 9\,\text{cm}$. **1**

(ii) What type of quadrilateral is $ABCE$? Give reasons. **2**

(iii) Show that $\Delta DXE \ ||| \ \Delta DBC$. **2**

(iv) Hence show that $BX.XE = 360$. **3**

Question 8. **[START A NEW PAGE]** **Marks**

(a) A function is defined by the following properties: **3**

$y = 0$ when $x = 1$; $\dfrac{dy}{dx} = 0$ when $x = -3,\ 1$ and 5;

and $\dfrac{d^2y}{dx^2} > 0$ for $x < -1$ and $1 < x < 3$.

Sketch a possible graph of the function.

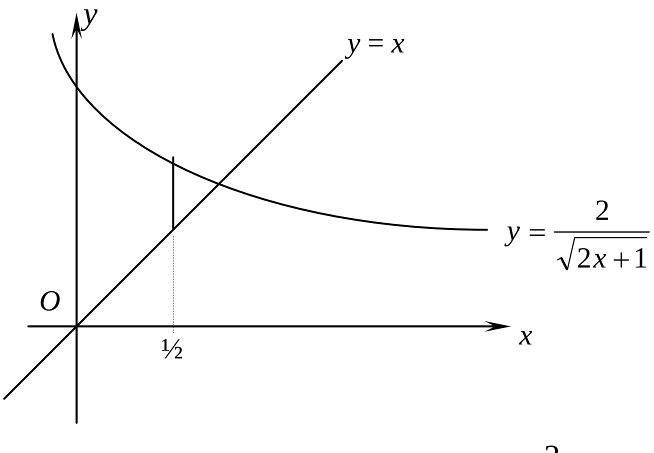
(b) Larsen begins his retirement with \$500 000 at the beginning of 2009.
The annual interest rate is 8% *p.a*. Interest is calculated annually on the balance at the beginning of the year and is added to the remaining balance.
Larsen plans to withdraw \$56 000 annually, with the first withdrawal at the end of 2009.
By letting A_n be the remaining balance after the *n*th withdrawal,

(i) Show that: $A_2 = 5\times10^5 R^2 - 5\cdot6\times10^4[1+R]$, where $R = 1\cdot08$. **2**

(ii) Hence deduce that: $A_n = 10^5[7 - 2R^n]$. **2**

(iii) Calculate during which year will Larsen's fund reach zero? **2**

(c)



Not to scale

The area bounded by the curves $y = \dfrac{2}{\sqrt{2x+1}}$, $y = x$, the lines $x = 0$ and $x = \dfrac{1}{2}$ **3**
is rotated about the *x*-axis.
Find the volume of the solid of revolution formed.

Question 9. **[START A NEW PAGE]** **Marks**

(a) Solve for x: $x^2 - 2|x| - 15 = 0$. **2**

(b) If $\log_a(xy^3) = 1$ and $\log_a(x^2 y) = 1$. What is the value of $\log_a(xy)$? **2**

(c) The rate at which carbon dioxide (CO_2) will be produced when conducting an experiment is given by $\dfrac{dV}{dt} = 0 \cdot 01(30t - t^2)$, where V cm^3 is the volume of the gas at t minutes.

(i) At what rate is the gas being produced 15 minutes after the experiment begins? **1**

(ii) How much carbon dioxide has been produced during this time? **2**

(d) The mass S grams of a substance decays at an instantaneous rate proportional to its mass present at time t years.

(i) Explain why the rate equation is given by: $\dfrac{dS}{dt} = -kS$, where k (> 0) is the decay rate constant of proportionality. **1**

(ii) Verify that $S = S_0 e^{-kt}$ is the general solution to the rate equation, where S_0 is the initial mass of the substance. **1**

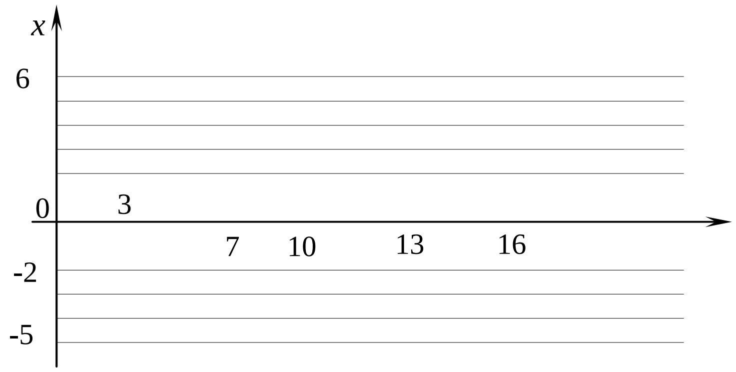
(iii) Show that the half-life for the substance is given by $\dfrac{\ln 2}{k}$ years **1**

(iv) If the mass present at time t_1 is S_1 and the mass at time t_2 is S_2, show that **2**

$$k = \frac{1}{t_2 - t_1}\ln\left(\frac{S_1}{S_2}\right), \text{ assuming } t_2 > t_1.$$

Question 10. **[START A NEW PAGE]** **Marks**

(a)



Not to scale

A particle is moving along a straight line according to the sketch of displacement x metres against time t seconds above.

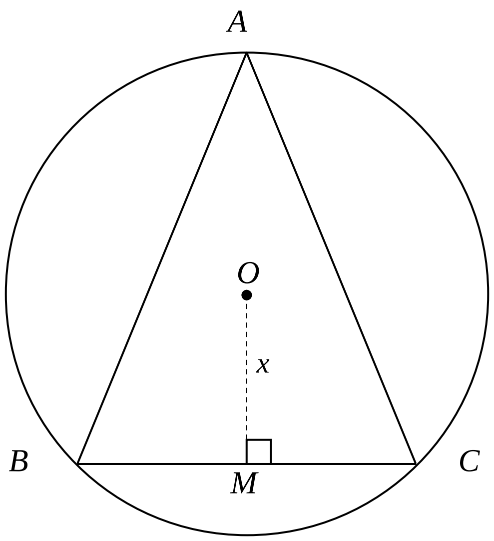
(i) When is the particle at rest? **1**

(ii) At what time does the particle have greatest speed (approximately)? **1**

(iii) Describe what happens to the particle as $t \to \infty$. **1**

(iv) Calculate the distance that the particle has eventually travelled. **2**

(b)



Not to scale

An isosceles triangle ABC with $AB = AC$ is inscribed in a circle centre O and of radius R units.

Given that $OM = x$ units, $OM \perp BC$ and M is the midpoint of BC,

(i) Show that the area of ΔABC, S square units, is given by: **2**

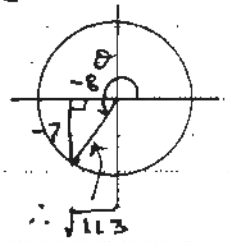
$$S = (R + x)\sqrt{R^2 - x^2}.$$

(ii) Hence show that the triangle with maximum area is an equilateral triangle. **5**

THE END ☹ 😐 ☺

2U MATHEMATICS TRIAL, 2009.
MARKING SCHEME

MATHEMATICS: Question 1

Suggested Solutions	Marks	Marker's Comments
1.(a) $\dfrac{e^3 - 2.1^2}{\sqrt{3.14+2.1}} = \dfrac{e^3 - 4.41}{\sqrt{5.24}} = 6.847890\ldots$ $= \underline{6.8}$ (2SF) ✓		$\boxed{1}$
(b) $\tan\theta = \frac{1}{8}$ $\cos\theta < 0 \Rightarrow 3^{RD}$ Quadrant  $\csc\theta = \dfrac{1}{\sin\theta} = \underline{-\dfrac{\sqrt{113}}{7}}$ $\therefore \sqrt{113}$		$\boxed{2}$
(c) $\lvert 4x - 15 \rvert \le 3$ $\therefore -3 \le 4x - 15 \le 3$ $12 \le 4x \le 18$ $\underline{3 \le x \le 9\frac{1}{2}}$ ✓✓		$\boxed{2}$
(d) $3y = \sin(2x - \frac{\pi}{4})$ $\therefore y = \frac{1}{3}\sin(2x - \frac{\pi}{4})$ Period $= \frac{2\pi}{2} = \underline{\pi}$ ‖ AMPLITUDE $= \underline{\frac{1}{3}}$		$\boxed{2}$
(e) SELLING = 130% Cost = \$67.60 $\therefore$ Cost $= \dfrac{\$67.60}{130} \times 100$ $= \underline{\$52}$		$\boxed{2}$
(f) $\tan\alpha = -0.5$ $0 < \alpha < \pi$ 2^{ND} Q. $\alpha = \pi - 0.4636\ldots$ $= 2.677945\ldots$ $\alpha = \underline{2.68}$ (2dp) ✓		$\boxed{1}$
(g) S = 11 21 31 41 51 61 12 22 32 42 52 62 13 23 33 43 53 63 14 24 34 44 54 64 ✓ 15 25 35 45 55 65 16 26 36 46 56 66 P(E = perfect sq digits) $= \underline{\dfrac{8}{36}}$ ✓ $= \dfrac{2}{9}$		$\boxed{2}$

MATHEMATICS: Question 2

Suggested Solutions | **Marks** | **Marker's Comments**

Q2(a) $m_{AC} = \dfrac{4\frac{1}{2} - 0}{0 - 5} = -\dfrac{4\frac{1}{2}}{5} = -\dfrac{9}{10}$

Eqn. of AC : $y = mx + b$

$y = -\frac{9}{10}x + 4\frac{1}{2}$

$10y = -9x + 45$

$\therefore 9x + 10y - 45 = 0$ qed

[2]

(b) $AC^2 = 5^2 + 4\frac{1}{2}^2$ (Pyth Thm)

$AC^2 = \frac{181}{4}$

$\therefore AC = \frac{\sqrt{181}}{2}$ units

[1]

(c) $x - 2y + 9 = 0$ — (1) : AB

$4x - y - 20 = 0$ — (2) : CB

$\therefore 8x - 2y - 40 = 0$ — (2a)

$7x - 49 = 0$

$\therefore x = 7$

subst in (2) $28 - y - 20 = 0$

$\therefore y = 8$

$\therefore B = (7, 8)$

[2]

(d) $m_{AB} = \frac{1}{2}$ from $x - 2y + 9 = 0$

$\tan\theta = \frac{1}{2}$ $\angle\theta = 26^\circ 34' \doteqdot 27^\circ$

[2]

(e) AC : $9x + 10y - 45 = 0$

B : $(7, 8)$

$\perp$ dist $= \left| \dfrac{9 \times 7 + 10 \times 8 - 45}{\sqrt{9^2 + 10^2}} \right| = \dfrac{98}{\sqrt{181}}$

$\therefore$ AREA $\Delta ABC = \frac{1}{2} \times AC \times \perp d.$

$= \frac{1}{2} \times \frac{\sqrt{181}}{2} \times \frac{98}{\sqrt{181}} = 24\frac{1}{2}\ u^2$

[2]

(f)

AC : $9x + 10y - 45 \geq 0$

BC : $4x - y - 20 \leq 0$

AB : $x - 2y + 9 \geq 0$

[3]

MATHEMATICS: Question 3

Suggested Solutions	Marks	Marker's Comments

Q3(a) (i) $y = (1+\ln x)^2$ ✓

$\frac{dy}{dx} = 2(1+\ln x) \times \frac{1}{x} = \frac{2(1+\ln x)}{x}$

$\boxed{2}$

(ii) $y = \frac{\sin x}{e^{3x}+1}$: $\frac{dy}{dx} = \frac{\cos x(e^{3x}+1) - \sin x \times 3e^{3x}}{(e^{3x}+1)^2}$ ✓✓

$\frac{dy}{dx} = \frac{(e^{3x}+1)\cos x - 3\sin x . e^{3x}}{(e^{3x}+1)^2}$

$\boxed{2}$

(b) (i) $\int (5x-1)^3 dx = \frac{1}{4\times5}(5x-1)^4 + c$ ✓✓

$= \frac{1}{20}(5x-1)^4 + c$

$\boxed{2}$

(ii) $\int_0^{\pi} \sec^2 \frac{x}{4}\, dx = 4\left[\tan \frac{x}{4}\right]_0^{\pi}$ ✓

$= 4[\tan \frac{\pi}{4} - \tan 0]$

$= 4[1-0] = 4$ ✓

$\boxed{2}$

(c) (i) $2x^2 - 6x - 7 = 0$

$\therefore \alpha+\beta = -\frac{b}{a} = -\frac{(-6)}{2} = 3$ ✓

$\boxed{1}$

(ii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= 3^2 - 2\times(-\frac{7}{2}) = 9+7$

$= 16$ ✓

(or) $2\alpha^2 - 6\alpha - 7 = 0$ cos $\alpha \rightarrow$ a root

$2\beta^2 - 6\beta - 7 = 0$

$2(\alpha^2+\beta^2) - 6(\alpha+\beta) - 14 = 0$

$2(\alpha^2+\beta^2) - 6\times3 \quad 14 = 0$

$\therefore \alpha^2+\beta^2 = \frac{32}{2} = 16$

$\boxed{2}$

(iii) As α is a root

$2\alpha^2 - 6\alpha - 7 = 0$

$2\alpha^2 - 6\alpha = 7$

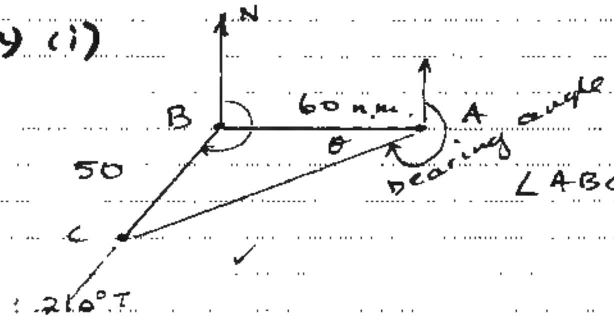
$\therefore \alpha^2 - 3x = \frac{7}{2} = 3\frac{1}{2}$ ✓

$\boxed{1}$

MATHEMATICS: Question 4

Suggested Solutions	Marks	Marker's Comments

Q 4(a) (i)



$$\begin{array}{r} 210^\circ \\ -90^\circ \\ \hline 120^\circ \end{array}$$

$\angle ABC = 120^\circ$ ✓ [1]

(ii) $AC^2 = 50^2 + 60^2 - 2\times 50 \times 60 \times \cos 120^\circ$

$= 9100$ ✓

$AC = 10\sqrt{91} = 95.3939 2014\ldots$

$\therefore \underline{AC = 95 \text{ n.miles}}$ (nearest nm) ✓ [2]

(iii) Let $\angle CAB = \theta^\circ$

$\dfrac{\sin\theta^\circ}{50} = \dfrac{\sin 120^\circ}{AC}$

$\sin\theta^\circ = \dfrac{50 \times \sin 120^\circ}{AC} = \begin{cases} 0.4539206\ldots \\ 0.455802\ldots \end{cases}$

$\angle\theta = \begin{cases} 27^\circ \text{ or } 153^\circ \\ 27^\circ 7' \text{ or } 152^\circ 53' \end{cases}$ but $120^\circ + 153^\circ > 180$

$\therefore \angle\theta = 27^\circ$ or $27^\circ 7'$ ✓

$\therefore$ Bearing of C from A is $243^\circ T$ / $242^\circ 53' T$ ✓ [2]

(b) $\displaystyle\lim_{x\to 16} \frac{x-16}{\sqrt{x}-4} = \lim_{x\to 16} \frac{(\sqrt{x}-4)(\sqrt{x}+4)}{\sqrt{x}-4}$ ✓

$= \displaystyle\lim_{x\to 16} (\sqrt{x}+4)$ as $x \neq 16$

$= \sqrt{16}+4 = \underline{8}$ ✓ [2]

or $\dfrac{x-16}{\sqrt{x}-4} \times \dfrac{\sqrt{x}+4}{\sqrt{x}+4}$

$= \dfrac{(x-16)(\sqrt{x}+4)}{x-16}$

$= \sqrt{x}+4$

(c) $y = 1 + \ln 2x$

$\dfrac{dy}{dx} = \dfrac{1}{2x} \times 2 = \dfrac{1}{x}$ ✓

Grad of Tangent at $x = \frac{1}{2}e$: $m_T = \dfrac{2}{e}$

" of Normal $m_N = -\dfrac{e}{2}$ ✓

Eqn. of Normal at $(\frac{1}{2}e, 2)$: $y - 2 = -\dfrac{e}{2}(x - \frac{1}{2}e)$ ✓

$\therefore \underline{y = -\frac{1}{2}ex + \frac{1}{4}e^2 + 2}$

$2ex + 4y - e^2 - 8 = 0$ [3]

(d) $4x^2 - 4x + 4y^2 + 24y + 21 = 0$

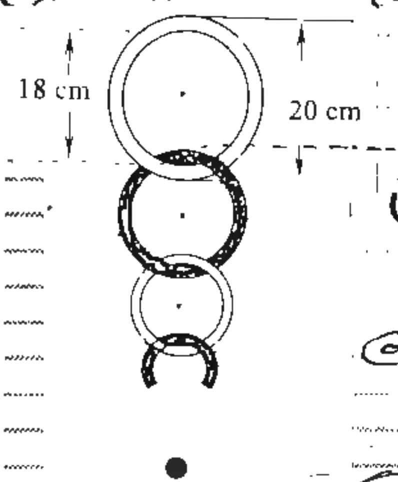
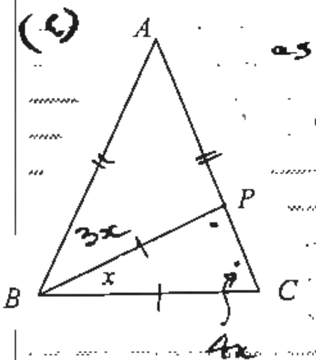
$x^2 - x + \frac{1}{4} + y^2 + 6y + 9 = -\frac{21}{4} + \frac{1}{4} + 9$

$-\frac{1}{2}$ $+3$

$(x - \frac{1}{2})^2 + (y+3)^2 = 4$ ✓

$\therefore$ centre is $\underline{(\frac{1}{2}, -3)}$ ✓ radius $= \underline{2}$ [2]

MATHEMATICS: Question 5

Suggested Solutions	Marks	Marker's Comments
Q5 (a) (i) (α) $P(E = W\ 1^{st}_{\$1000}) = \frac{1}{1000}$ ✓		1
(β) $P(E = \text{at least } \$500) = P(\$1000) + P(\$500) = \frac{2}{1000}$ ✓		1
(γ) $P(E = \text{no prize}) = P(\$0) = 1 - \frac{3}{1000} = \frac{997}{1000}$	✓	1
(ii) 2 tickets $P(E \geq \$500) = 1 - P(\text{no prize in 2 tickets})$ $= 1 - \frac{998}{1000} \times \frac{997}{999} = 0.00\dot{3}99\dot{7}$		2
(b) (i) The top of the 1^{st} ring is 20cm above its bottom!! this bottom is 2cm below the top of 2^{ND} ring so it is 19cm − 2cm = 17cm above the bottom of 3^{RD} ring.		1
 (ii) Total Dist. $= 20 + 17 + 16 + \ldots + 2 + 1$ $= 20 + \frac{17}{2}(1 + 17) = 20 + 17 \times 9$ $= \underline{173\text{cm}}$ or $20 + 19 + \ldots + 4 + 3 - 2 \times 17$ $= 20 + 19 + \ldots + 2 + 1 - 3 - 34$ $= \frac{20 \times 21}{2} - 37 = \underline{173\text{cm}}$ or Distance from tops $= 18 + 17 + \ldots + 2 + 3$ $= 18 + 17 + \ldots + 2 + 1 + 2$		sum of diameters − 2cm overlaps of 17 = 173cm 3
(c)  as $\angle ABP = 3x$ 1. $\angle ACB = 4x$ (equal angles opposite equal sides) 2. $\angle CPB = 4x$ (equal angles opposite equal sides) 3. $\therefore 4x + 4x + x = \pi$ (Angle sum of $\triangle BPC$ is π) ✓ $9x = \pi$ $x = \underline{\frac{\pi}{9}}$ ✓ $\angle CBP = 20^\circ$ (in degrees)		AB = AC ✓ BP = BC 3

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MATHEMATICS: Question 6

Suggested Solutions	Marks	Marker's Comments

26(a) $LHS = \dfrac{1+\tan^2 A}{\operatorname{cosec}^2 A} = \dfrac{\sec^2 A}{\operatorname{cosec}^2 A}$ ✓

Approach I:

$= \dfrac{1}{\cos^2 A} \div \dfrac{1}{\sin^2 A}$

$= \dfrac{\sin^2 A}{\cos^2 A}$ ✓

$= \tan^2 A$ q.e.d.

II: $\dfrac{1+\tan^2 A}{\operatorname{cosec}^2 A} = \sin^2 A\left(1+\dfrac{\sin^2 A}{\cos^2 A}\right)$

$= \dfrac{s^2}{c^2}(c^2+s^2)$

$= \dfrac{s^2}{c^2} = \tan^2 A$

[2]

(b)

x	1	1·25	1·5	1·75	2	2·25	2·5
$f(x)$	3·4	2·2	0·4	1·9	−2·7	1·3	2
	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7

$h = \dfrac{2\cdot5-1}{6} = \dfrac{1\cdot5}{6} = \dfrac{1}{4} = 0\cdot25$ ✓

$\int_1^{2\cdot5} f(x)\,dx \doteq A_3 \doteq \dfrac{h}{3}\left[Y_1+Y_7+4\times(Y_2+Y_4+Y_6)+2(Y_3+Y_5)\right]$

$= \dfrac{0\cdot25}{3}\left[5\cdot4 + 4\times(2\cdot2+1\cdot9+1\cdot3)+2(0\cdot4+-2\cdot7)\right]$

$= \dfrac{0\cdot25}{3}\left[5\cdot4 + 4\times5\cdot4 + 2\times(-2\cdot3)\right]$

$= \dfrac{0\cdot25}{3}\left[5\cdot4+21\cdot6-4\cdot6\right] = \dfrac{0\cdot25}{3}\times22\cdot4$

$= 1\cdot8\dot6$

$= 1\cdot9$ (1 dp)

[3]

(c) $g(x) = x^3-6x^2+5 = (x-1)(x^2-5x-5)$

(i) $g'(x) = 3x^2-12x = 3x(x-4)$

$g''(x) = 6x-12 = 6(x-2)$

For SPs to occur $g'(x)=0$

ie $3x(x-4)=0$

$x = 0$ or 4

$\therefore g(0) = 5$ or -27

$\therefore$ SPs are $(0,5)$ and $(4,-27)$

TEST: at $x=0$: $g''(0) = -12 < 0$ $\therefore$ concave downwards $\therefore$ Rel. max TP at $(0,5)$

at $x=4$: $g''(4) = 12 > 0$ $\therefore$ concave upwards $\therefore$ Rel. min TP at $(4,-27)$

[3]

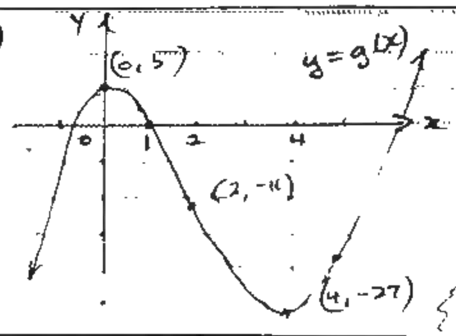
(ii) change in concavity can be at $g''(x)=0$ P.O.I.

so at $x = 2$ $\therefore (2,-11)$ ✓

x	1	2	3
$g''(x)$	−6	0	6

[1]

(iii) [2]



ignore $x=\dfrac{5\pm\sqrt{45}}{2}$

(iv) $-3 \le x \le 5$

$g(-3) = -76$

$g(5) = -20$

$\therefore$ From sketch $y=g(x)$

Absolute min is -76 at $x=-3$ ✓

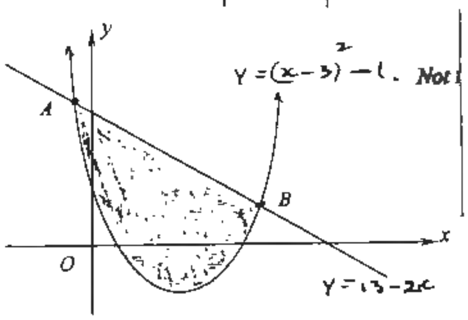
[1]

MATHEMATICS: Question 7.

Suggested Solutions	Marks	Marker's Comments

Q7(a) $y = (x-3)^2 - 1$

$y = 13 - 2x$



(i) Points of intersection

$(x-3)^2 - 1 = 13 - 2x$

$x^2 - 6x + 9 - 1 = 13 - 2x$

$x^2 - 4x - 5 = 0$

$(x+1)(x-5) = 0$

$\therefore x = -1$ or 5

$\underline{x_A = -1} \quad \underline{x_B = 5}$

$\boxed{2}$

(ii)

Area $= \int_{-1}^{5} (Y_U - Y_L)\,dx$

$= \int_{-1}^{5} 13 - 2x - \{(x-3)^2 - 1\}\,dx$

$= \int_{-1}^{5} 14 - 2x - (x-3)^2\,dx \qquad = \int_{-1}^{5} 5 + 4x - x^2\,dx$

$= \left[14x - x^2 - \frac{1}{3}(x-3)^3\right]_{-1}^{5} \qquad = \left[5x + 2x^2 - \frac{1}{3}x^3\right]_{-1}^{5}$

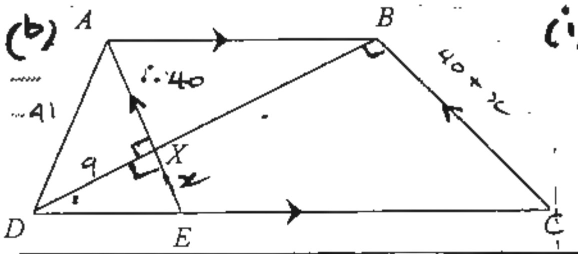
$= \left[\left(70 - 25 - \frac{8}{3}\right) - \left(-14 - 1 + \frac{64}{3}\right)\right] \qquad = 78 - 42$

$= 42\frac{1}{3} - 6\frac{1}{3} \qquad = 36$

Area $= 36$ sq. units

$\boxed{2}$

(b)



(i) $41^2 = 9^2 + AX^2$ (Pyth. Thm.)

$\underline{AX = 40 \text{ cm}}$

$\boxed{1}$

(ii) As $\angle DXE = \angle DBC = 90^\circ$ and are corresponding angles ✓

$\therefore$ AE ∥ CB and as AB ∥ DC

$\therefore$ ABCE is a $\underline{\text{Parallelogram}}$ ✓ (two pair of parallel sides)

$\boxed{2}$

(iii) In Δs DXE and DBC

1. $\angle XDE = \angle BDC$ (common) ✓
2. $\angle DXE = \angle DBC = 90^\circ$ (data)

$\therefore \Delta DXE \parallel\!\mid \Delta DBC$ (equiangular / matching angles equal) ✓

$\boxed{2}$

(iv) $AE = 40 + XE$

but $BC = AE$ (opp. sides of ∥ogram are equal) ✓

$\therefore \frac{BC}{XE} = \frac{DB}{DX}$ (Corresponding sides in similar triangles are in the same ratio) ✓

or $\frac{XE}{BC} = \frac{DX}{DB}$

$\frac{XE}{40 + XE} = \frac{9}{9 + BX}$

i.e. $\frac{40 + XE}{XE} = \frac{9 + XB}{9}$ ✓

$360 + 9XE = 9XE + XB.XE$

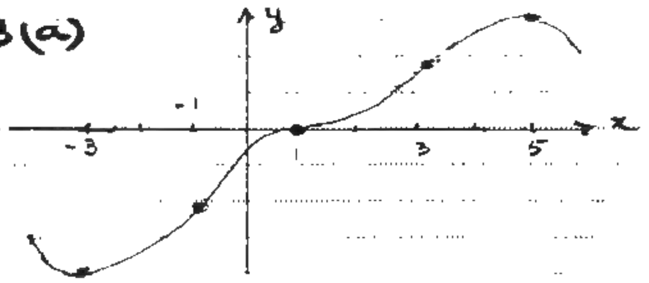
$\therefore BX.XE = 360$ q.e.d.

$\boxed{3}$

MATHEMATICS: Question 8

Suggested Solutions	Marks	Marker's Comments

Q8(a)



Possible sketch.

Rel min TP $x=-3$ ✓
Rel max TP $x=5$
P.o.I. at $x=-1, 3$ ✓
HPoI at $(1,0)$ ✓

[3]

(b) $r = 8\% = 0.08$

$\therefore R = 1+\frac{r}{100} = 1.08$ given

(i)

Begin 2010 $A_1 = 500\,000 \times R - 56\,000$ ✓

Begin 2011 $A_2 = A_1R - 56\,000$

$= (5\times10^5 R - 5.6\times10^4)R - 5.6\times10^4$ ✓

$= 5\times10^5R^2 - 5.6\times10^4(1+R)$ q.e.d.

[2]

(ii) so Begin 2012

$A_3 = 5\times10^5 R^3 - 5.6\times10^4[1+R+R^2]$

$\therefore$ After n years

$A_n = 5\times10^5R^n - 5.6\times10^4[1+R+R^2+\dots+R^{n-1}]$ ✓

A G.Series "$a=1$, $r=R$"

$= 5\times10^5R^n - 5.6\times10^4\left[\frac{R^n-1}{R-1}\right]$

$= 5\times10^5R^n - 5.6\times10^4\frac{[R^n-1]}{0.08}$; $R-1 = 1.08-1$

$\frac{5.6}{0.08} = 70$

$= 5\times10^5R^n - 7\times10^5(R^n-1)$ ✓

$= 5\times10^5R^n - 7\times10^5R^n + 7\times10^5$

$\therefore A_n = 10^5[7-2R^n]$ q.e.d.

[2]

(iii) when $A_n = 0$

$\therefore 7-2R^n = 0$

$1.08^n = \frac{7}{2} = 3.5$ ✓

$n = \frac{\ln 3.5}{\ln 1.08} = 16.27788\dots$

$\therefore$ Funds runs out during 17th year ie. during 2025 ✓ (but actually at end of 2025 ✓)

[2]

(c) Vol $= \pi\int_0^{\frac{1}{2}} y_u^2\,dx - \pi\int_0^{\frac{1}{2}} y_L^2\,dx$

$= \pi\int_0^{\frac{1}{2}}\left(\frac{4}{2x+1} - x^2\right)dx$ ✓

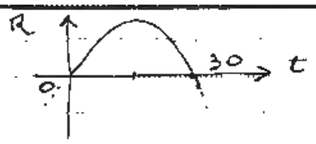
$= \pi\left[2\ln(2x+1) - \frac{1}{3}x^3\right]_0^{\frac{1}{2}}$ ✓

$= \pi\left[2\ln 2 - \frac{1}{24} - 0\right]$

$\underline{\text{Vol} = \pi\left(\ln 4 - \frac{1}{24}\right)\text{ units}^3}$ ✓

[3]

MATHEMATICS: Question 9

Suggested Solutions	Marks	Marker's Comments
Q9 (a) $x^2 - 2\|x\| - 15 = 0$ Let $u = \|x\|$ $u^2 = \|x\|^2 = x^2$ $\Rightarrow u^2 - 2u - 15 = 0$ $(u+3)(u-5) = 0$ $\therefore u = \|x\| = -3$ or 5 ✓ $\|x\| = -3$ NO SOLUTIONS POSSIBLE $\|x\| = 5 \Rightarrow \underline{x = \pm 5}$ ✓	$\|x\| \geq 0$	$\boxed{2}$
(b) $\log_a(xy^3) = 1 \Rightarrow \log_a x + 3\log_a y = 1$ — (1) $\log_a(x^2y) = 1 \Rightarrow 2\log_a x + \log_a y = 1$ — (2) $2\times(1)$ $\quad 2\log_a x + 6\log y = 2$ — (1A) $0 \quad 5\log y = 1$ $\log_a y = \frac{1}{5}$ $\therefore \log_a x = \frac{2}{5}$ $\therefore \log(xy) = \log x + \log y$ $= \frac{2}{5} + \frac{1}{5}$ $\underline{\log_a(xy) = \frac{3}{5}}$	OR	$xy^3 = a$ — (1) $x^2y = a$ — (2) $\frac{y^2}{x} = 1$ $x = y^2$ — (3) $(1) \Rightarrow y^5 = a$ $y = a^{\frac{1}{5}}$ $\therefore x = a^{\frac{2}{5}}$ $\boxed{2}$
(c) $\frac{dV}{dt} = 0.01(30t - t^2)$  (i) when $t = 15$ Rate $= 0.01(30\times 15 - 15^2)$ $\underline{\text{Rate} = 2.25 \text{ cm}^3/\text{min}}$ ✓		$\boxed{1}$
(ii) $V = \int 0.01(30t - t^2)\,dt$ $V = 0.01(15t^2 - \frac{1}{3}t^3) + C$ ✓ Let $V = 0$ at $t = 0$ gives $C = 0$ so $t = 15$ $V = 0.01(15^3 - \frac{1}{3}.15^3) = 22.5$ Volume produced is $\underline{22.5 \text{ cm}^3}$	✓	$\boxed{2}$
(d) PTO		

MATHEMATICS : Question 9 CONT.

Suggested Solutions	Marks	Marker's Comments
Q9 (d) (i) Rate of decay is proportional to mass present (data) ✓ $\therefore \frac{dS}{dt} \propto -S$ so $\frac{dS}{dt} = -kS$, where k is ...		1
(ii) $S = S_0 e^{-kt}$ VERIFY LHS: $\frac{dS}{dt} = S_0 \times -ke^{-kt}$ $= -kS_0 e^{-kt}$ RHS: $-kS = -k \times S_0 e^{-kt}$ ✓ $\therefore$ LHS = RHS $\therefore S = S_0 e^{-kt}$ satisfies rate equation.		1
(iii) when $S = \frac{1}{2} S_0$ as $t = 0$ $S = S_0$ $\therefore \frac{1}{2} S_0 = S_0 e^{-kt}$ $e^{-kt} = \frac{1}{2}$ $-kt = \ln\frac{1}{2} = -\ln 2$ ✓ $t_{\frac{1}{2}} = \frac{\ln 2}{k}$ q.e.d.		$e^{kt} = \frac{2}{1}$ $kt = \ln 2$ $t = \frac{\ln 2}{k}$ 1
(iv) E_1: $S_1 = S_0 e^{-kt_1}$ — (1) E_2: $S_2 = S_0 e^{-kt_2}$ — (2) $t_2 > t_1$ $\therefore \frac{S_1}{S_2} = e^{-kt_1 + kt_2} = e^{-k(t_1 - t_2)}$ ✓ $\ln\left(\frac{S_1}{S_2}\right) = -k(t_1 - t_2) \equiv k(t_2 - t_1)$ ✓ $\therefore k = \frac{1}{t_2 - t_1} \ln\left(\frac{S_1}{S_2}\right)$ q.e.d.		2
(v) —		

MATHEMATICS: Question 10

Suggested Solutions	Marks	Marker's Comments

Q10 (a).

(i) At rest when $v = \frac{dx}{dt} = 0$

$\therefore$ at $t = 3$ and 10 (3 and 10 seconds) $\boxed{1}$

(ii) At greatest speed $\frac{dv}{dt} = 0$ and $\frac{dx}{dt} = \tan\theta$

$\therefore$ at approx 7 seconds ($t=7$) $\boxed{1}$

(iii) the particle approaches 0 from $x > 0$ at ever decreasing speed ($v < 0$ and $\ddot{x} > 0$) $\boxed{1}$

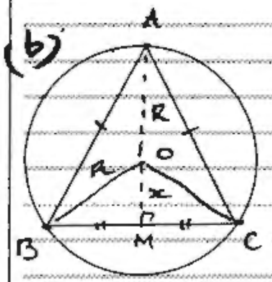
(iv)

Total distance = 3 + . + 6
= 20 m.

$-2 \to -5$: 3 m
$-5 \to 6$: 11 m
$6 \to 0$: 6 m

$\boxed{2}$

(b)



(i) Area of ΔABC $S = \frac{1}{2} BC \times h = \frac{1}{2} \times BC \times AM$.

$R^2 = x^2 + BM^2$, (Pyth. thm)

$BM^2 = R^2 - x^2$

$BM = \sqrt{R^2 - x^2}$

$\therefore BC = 2BM = 2\sqrt{R^2 - x^2}$

$h = AM = x + R$

$\therefore S = \frac{1}{2} \times 2\sqrt{R^2 - x^2} \times (R + x)$

i.e. $S = (R + x)\sqrt{R^2 - x^2}$ $\quad 0 \le x \le R$. $\boxed{2}$

(ii) $\frac{dS}{dx} = 1.\sqrt{R^2 - x^2} + (R + x)\frac{1}{2}(R^2 - x^2)^{-\frac{1}{2}} \cdot -2x$

$= \sqrt{R^2 - x^2} - \frac{x(R + x)}{\sqrt{R^2 - x^2}}$

$= \frac{R^2 - x^2 - Rx - x^2}{\sqrt{R^2 - x^2}}$

$\frac{dS}{dx} = \frac{R^2 - Rx - 2x^2}{\sqrt{R^2 - x^2}}$ ✓ 1

For possible max/min values of S to occur $\frac{dS}{dx} = 0$

so $R^2 - Rx - 2x^2 = 0$ only

$2x^2 + Rx - R^2 = 0$

$(2x - R)(x + R) = 0$

$x = -R$ or $\frac{1}{2}R$ 1

so $x = \frac{1}{2}R$ ✓ as $x > 0$ as $0 \le x \le R$

$\therefore S = \frac{3}{2}R\sqrt{R^2 - \frac{R^2}{4}} = \frac{3\sqrt{2}}{4}R^2$

continued →

MATHEMATICS: Question 10 CONT.

Suggested Solutions	Marks	Marker's Comments

TEST nature at $x = \frac{1}{2}R$

METHOD 1

$\frac{R^2 - Rx - 2x^2}{\sqrt{R^2 - x^2}} = \frac{(R-2x)(R+x)}{\sqrt{R^2-x^2}}$

x	0	$\frac{1}{2}R$	$\frac{1}{\sqrt{2}}R$ or ...
$\frac{ds}{dx}$	R (+)	0	$\frac{-R^2/2}{\sqrt{R^2/2}}$ (−)
	/	—	\

$\therefore$ A Rel max TP at $x = \frac{1}{2}R$ ✓

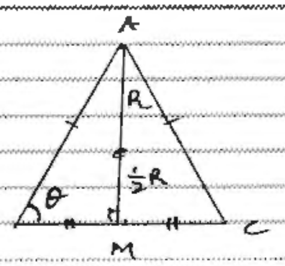
METHOD 2: $x = 0 \quad S = R^2$

$x = \frac{R}{2} \quad S = \frac{3}{4}\sqrt{2}R^2 > R^2$

$x = R \quad S = 0$

and since S is continuous for $0 \le x \le R$
so Rel max TP is the absolute max at $x = \frac{1}{2}R$

Now



$BM = \sqrt{R^2 - x^2} = \sqrt{R^2 - \frac{R^2}{4}} = \frac{R\sqrt{3}}{2}$

$\therefore BC = R\sqrt{3}$

$\tan\theta = \frac{R + \frac{1}{2}R}{\frac{R\sqrt{3}}{2}} = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\therefore \theta = 60°$ ✓

so $\angle C = \angle B = 60°$ (equal angles opposite equal sides) (data)

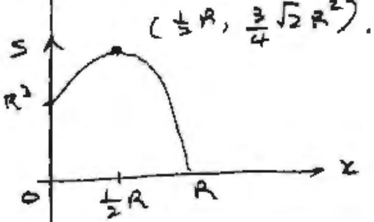
$\therefore \angle A = 60°$

$\therefore$ max area when $x = \frac{1}{2}R$ and $\triangle ABC$ is equilateral.
qed

Marker's Comments:

$x = \cdot 7R \quad \frac{ds}{dx} = \frac{-0\cdot6 \times 1\cdot7R^2}{\sqrt{0\cdot51R^2}} < 0$

$x = 0\cdot9R \quad \frac{ds}{dx} = \frac{-0\cdot2 \times 1\cdot9R^2}{\sqrt{0\cdot19R^2}} < 0$



Aw 2: For A proof that $\triangle ABC$ is equilateral when $x = \frac{1}{2}R$

$\boxed{5}$

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